

Dynamics of Particles Around a Regular Black Hole with Nonlinear Electrodynamics

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Abstract

We investigate the dynamics of a charged particle being kicked off from its circular orbit around a regular black hole by an incoming massive particle in the presence of magnetic field. The resulting escape velocity, escape energy and the effective potential is analyzed. It is shown that the presence of even a very weak magnetic field helps the charged particles in escaping the gravitational field of the black hole. Moreover the effective force acting on the particle visibly reduces with distance. Thus particle near the black hole will experience higher effective force as compared to when it is far away.

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1 Introduction

Dynamics of particles is an important and challenging topic in the astrophysical studies of black holes. Partly because if the particles are accreted by black holes than black holes gain mass, and possibly black holes hurl particles away at high relativistic speeds due to the angular momentum barrier. Moreover high energy charged particles can also escape by the collision with other particles from their stable orbits around black holes and move in the electromagnetic fields under the Lorentz force law. Particles in the surroundings of black holes are significantly influenced by the strong gravitational pull.

The observational evidences [1, 2] indicate that the magnetic field arises due to plasma which may exist in the surrounding of black holes in the form of an accretion disk or a charged gas cloud [3, 4]. The relativistic motion of charged particles in the accretion disk generates the magnetic field. This magnetic field also leads to the generation of gigantic jets along the magnetic axes. Since general relativity (GR) predicts that all kinds of fields produce gravitational effects via curvature of spacetime, it is assumed that the magnetic field is relativity weak such that it does not affect the geometry of the black hole yet it may affect the motion of charged particles [5, 6]. Due to the presence of strong gravitational and electromagnetic fields, the motion of charged particles is in general chaotic and unpredictable. Moreover the corresponding orbits are unstable except the innermost-stable-circular orbit.

If the collision of two particles occur near the event horizon of black hole than high center of mass energy is produced. Bañados, Silk and West (BSW) [7] have proposed the mechanism for the collision of two particles falling from rest at infinity towards the Kerr black hole and determined that the center of mass (CM) energy in the equatorial plane may be highest in the case of a fast rotating black hole or the extremal black hole. Lake [8] found that the CM energy of the particles at the inner horizon of Kerr black hole. Further, Wei et al [9] investigated that the CM energy of the collision of the particles around Kerr-Newmann black hole. A general review of the collision mechanism is found in [10]. The CM energy of the collision of particles near the horizons of Kerr-Taub-NUT black hole [11], Kerr-Newman-Taub-NUT black hole [12], Plebanski-Demianski black hole [13], charged dilaton black hole [14], cylindrical black hole [15] have been investigated. The BSW effect has been studied for different black hole [16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29].

Recently, the dynamics of particles moving around weakly magnetized

black holes have been investigated. Motion of a charged particle near weakly magnetized Schwarzschild black hole has been analyzed in [30, 31, 32]. The chaotic motion of a charged particles around Kerr black hole near magnetic field has also been investigated in [33, 34, 35, 36, 37, 38, 39]. The Circular motion of charged particles around Reissner-Nordstrom black hole has been analyzed by Pugliese et al [40]. The particle dynamics around Riessner-Nordstrom black hole with magnetic field has been studied by Majeed et al [41]. The dynamics of a charged particle around a slowly rotating Kerr black hole immersed in magnetic field has been discussed by Hussain et al [42]. Also the dynamics of particles around a Schwarzschild-like black hole in the presence of quintessence and magnetic field has also been discussed by Jamil et al. [43]. In the present work, we want to investigate the dynamics of particles around a regular black hole surrounded by external magnetic field.

The organization of the work is as follows: In section **2**, we review the regular black hole metric with non-linear electromagnetic source and study the effective potential and energy of particle. Section **3** deals with the dynamics of charged particle. In section **4**, we investigate the center of mass energy of two colliding particles. In section **5**, we find the effective force and analyze the graphical representations. Finally, the results and conclusions of the work are presented in section **6**. We adopt the units $c = G = 1$ and the metric signature $(+, -, -, -)$.

2 Regular Black Hole With Non-linear Electromagnetic Source

It is well-known that linear or Maxwell electrodynamics leads to curvature singularity and multiple horizons in Reissner-Nordström spacetime. This problem of curvature singularity gets fixed when quadratic (or generalized) Maxwell tensor is used. Regular black holes are solutions of Einstein's field equations that have horizon(s) and but no curvature singularity. To avoid the singularity, Bardeen [44] proposed the concept of regular black hole, dubbed as Bardeen black hole and subsequently, another type of regular black hole (Hayward black hole) was discovered [45]. Another kind of regular black hole is Ayon-Beato-Garcya (ABG) black hole [46]. The regular black holes [47, 48] have the properties that their metrics as well as their curvature invariants are regular everywhere. This type of black holes violates the strong energy

condition somewhere in the spacetime; however, some of these solutions satisfy the weak energy condition (WEC) everywhere [49, 50]. Bronnikov [51] has even proved a theorem which asserts that the existence of electrically charged, static, spherically symmetric solutions with a regular center is forbidden, while the existence of the solutions with magnetic charges is feasible. In this connection, it is pertinent to study magnetically charged regular black holes for particle acceleration and particle's escape. Several regular black hole solutions have been found by coupling gravity to nonlinear electrodynamics [47, 48] theories.

A static spherically symmetric regular charged black hole is given by [48]

$$ds^2 = f(r)dt^2 - \frac{dr^2}{f(r)} - r^2(d\theta^2 + \sin^2\theta d\phi^2), \quad (1)$$

where

$$f(r) = 1 - \frac{2M}{r} \left(\frac{2}{\exp(\frac{q^2}{Mr}) + 1} \right). \quad (2)$$

where M and q are respectively the mass and charge of the black hole. Notice that $r = 0$ is no more a curvature singularity here while the horizon(s) exist at $f(r_h) = 0$. It is important to mention that in literature, a variety of static and non-static regular black holes have been discussed while we have chosen one of them to study particle dynamics. We expect that the particle dynamics around different static regular black holes as investigated here, should be generic.

In order to study the motion of particles in the background of a regular black hole, we employ the Lagrangian dynamics and use the following Lagrangian

$$\mathcal{L} = f(r)\dot{t}^2 - \frac{\dot{r}^2}{f(r)} - r^2(\dot{\theta}^2 + \sin^2\theta\dot{\phi}^2). \quad (3)$$

The overdot denotes derivative with respect to proper time. It is obvious from Eq. (3) that Lagrangian is independent of the time t and the ϕ coordinates explicitly but not of their derivatives, hence it leads to corresponding symmetry generators (also called Killing vectors). The Killing vector fields \mathbf{X} under which the spacetime (1) remains invariant (i.e. $\mathbf{X}\mathcal{L} = 0$) are

$$\mathbf{X}_0 = \frac{\partial}{\partial t}, \quad \mathbf{X}_1 = \frac{\partial}{\partial \phi}, \quad \mathbf{X}_2 = \cos\phi \frac{\partial}{\partial \theta} - \cot\theta \sin\phi \frac{\partial}{\partial \phi},$$

Table 1: Conservation Laws

Generator	First integrals
\mathbf{X}_0	$E = f(r)\dot{t}$
\mathbf{X}_1	$-L_z = r^2 \sin^2 \theta \dot{\phi}$
\mathbf{X}_2	$L_1 = r^2 \left(\cos \phi \dot{\theta} - \cot \theta \sin \phi \dot{\phi} \right)$
\mathbf{X}_3	$L_2 = r^2 \left(\sin \phi \dot{\theta} + \cot \theta \cos \phi \dot{\phi} \right)$

$$\mathbf{X}_3 = \sin \phi \frac{\partial}{\partial \theta} + \cot \theta \cos \phi \frac{\partial}{\partial \phi}. \quad (4)$$

The conservation laws corresponding to the symmetries are given in Table 1. In this table, E , L_z , L_1 and L_2 indicate total energy, azimuthal angular momentum and angular momenta, respectively. If the motion of particles is considered in the equatorial plane than the \mathbf{X}_2 and \mathbf{X}_3 become irrelevant.

The total specific angular momentum of the particle is defined as [1]

$$L^2 \equiv r^4 \dot{\theta}^2 + \frac{L_z^2}{\sin^2 \theta} = r^2 v_\perp^2 + \frac{L_z^2}{\sin^2 \theta}. \quad (5)$$

By using the normalization condition for four-velocity and solving for \dot{r}^2 , we obtain

$$\dot{r}^2 = f^2(r)\dot{t}^2 - f(r) \left(1 + r^2 \dot{\theta}^2 + r^2 \sin^2 \theta \dot{\phi}^2 \right). \quad (6)$$

Due to spherical symmetry, all $\theta = \text{constant}$ planes will be equivalent to the equatorial plane $\theta = \frac{\pi}{2}$, hence using the later value and the conservation laws we get

$$\dot{r}^2 = E^2 - f(r) \left(1 + \frac{L_z^2}{r^2} \right). \quad (7)$$

For the particle moving in a circular orbit around the black hole, $\dot{r} = 0$, Eq.(7) implies

$$E^2 = f(r) \left(1 + \frac{L_z^2}{r^2} \right) \equiv U_{\text{eff}}(r), \quad (8)$$

where U_{eff} is the effective potential. It suggests that at the horizon(s), the total energy and/or the effective potential will vanish. Particles with different

energy and angular momentum will move under entirely different potential energy curves as are depicted below.

Differentiating the effective potential U_{eff} with respect to r and setting it to zero, we obtain

$$L_z^2 = \frac{2r^2 \left(q^2 e^{\frac{q^2}{Mr}} - M r e^{\frac{q^2}{Mr}} - M r \right)}{r^2 (1 + e^{\frac{q^2}{Mr}})^2 + 2 \left(3 M r e^{\frac{q^2}{Mr}} - q^2 e^{\frac{q^2}{Mr}} + 3 M r \right)}. \quad (9)$$

It represents the critical angular momentum that a particle carry in an orbit where the effective potential has an extremum (i.e. maximum or minimum). Hence, the value of E^2 takes the form

$$E^2 = \frac{r^3 (e^{\frac{q^2}{Mr}} + 1)^3 - 16 M^2 r (e^{\frac{q^2}{Mr}} + 1)}{r^3 (e^{\frac{q^2}{Mr}} + 1)^3 + 2 r (e^{\frac{q^2}{Mr}} + 1) (3 M r (e^{\frac{q^2}{Mr}} + 1) - q^2 e^{\frac{q^2}{Mr}})}. \quad (10)$$

After collision, the energy of the particle takes the form

$$E_n^2 = f(r) \left(1 + \frac{(L_z + r v_\perp)^2}{r^2} \right). \quad (11)$$

For the particle to escape from the ISCO, we require the escape energy to be greater after collision than the total energy before collision. Observe the presence of an extra term $r v_\perp$ in (11) due to collision. Solving Eq.(11) for v_\perp , we have

$$v_\perp = \sqrt{\frac{E_n^2 - f(r)}{f(r)}} - \frac{L_z}{r}. \quad (12)$$

which is the minimum escape velocity for general values of E , $f(r)$ and L_z .

3 Charge particle motion in the presence of magnetic field

In this section, we adopt the formalism from the literature [32, 43]. The Lagrangian of a particle of mass m carrying an electric charge q takes the form

$$\mathcal{L} = g_{\mu\nu} u^\mu u^\nu + \frac{q}{m} A_\mu u^\mu, \quad (13)$$

where A_μ is the four vector potential for the electromagnetic field. Considering the presence of an axially symmetric magnetic field with strength B around black hole, we obtain new constants of motions for the charged particle as

$$E = f(r)\dot{t}, \quad L_z = r^2 \sin^2 \theta (\dot{\phi} + B), \quad (14)$$

Using the Eurler-Lagrange equation for r , we get

$$\ddot{r} = \frac{1}{2} \left(f(r)(2r\dot{\theta}^2 + 2r \sin^2 \theta \dot{\phi}^2) - f'(r)f(r)\dot{t}^2 - \frac{\dot{r}^2 f'(r)}{f(r)} \right), \quad (15)$$

where

$$f(r) = 1 - \frac{2M}{r} \left(\frac{2}{e^{\frac{q^2}{Mr} + 1}} \right), \quad \dot{t} = \frac{E}{f(r)}, \quad \dot{\phi} = \frac{L_z}{r^2 \sin^2 \theta} - B. \quad (16)$$

Using the value of \dot{t} and $\dot{\phi}$ from Eq.(16) in the Lagrangian (13), we get the energy and corresponding effective potential as follows

$$E^2 = \dot{r}^2 + r^2 \dot{\theta}^2 f(r) + f(r) \left(1 + r^2 \sin^2 \theta \left(\frac{L_z}{r^2 \sin^2 \theta} - B \right)^2 \right), \quad (17)$$

$$U_{\text{eff}} = f(r) \left(1 + r^2 \sin^2 \theta \left(\frac{L_z}{r^2 \sin^2 \theta} - B \right)^2 \right). \quad (18)$$

In order to integrate the dynamical equations, we need to make these equations dimensionless by using the the following transformations [42, 43, 52]

$$2M = r_d, \quad \sigma = \frac{\tau}{r_d}, \quad \rho = \frac{r}{r_d}, \quad l = \frac{L_z}{r_d}, \quad b = Br_d, \quad q = \frac{\alpha}{r_d}. \quad (19)$$

Using the transformation given in Eq.(19), the equation of motion, energy and effective potential acquire the form

$$\begin{aligned} \ddot{\rho} &= \frac{1}{2} \left(f(\rho)(2\rho\dot{\theta}^2 + 2\rho \sin^2 \theta \dot{\phi}^2) - f'(\rho)f(\rho)\dot{t}^2 - \frac{\dot{\rho}^2 f'(\rho)}{f(\rho)} \right), \\ E^2 &= \dot{\rho}^2 + \rho^2 \dot{\theta}^2 f(\rho) + f(\rho) \left(1 + \rho^2 \sin^2 \theta \left(\frac{l}{\rho^2 \sin^2 \theta} - b \right)^2 \right), \\ U_{\text{eff}} &= f(\rho) \left(1 + \rho^2 \sin^2 \theta \left(\frac{l}{\rho^2 \sin^2 \theta} - b \right)^2 \right). \end{aligned} \quad (20)$$

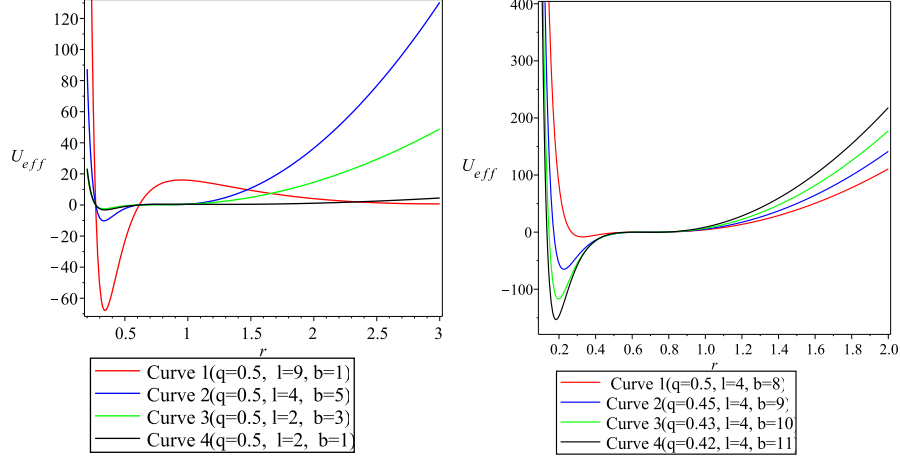


Figure 1: The plot of effective potential (U_{eff}) versus r . In the left panel, we fix $q = 0.5$ and vary l and b . In the right panel, we fix $l = 4$ and vary q and b .

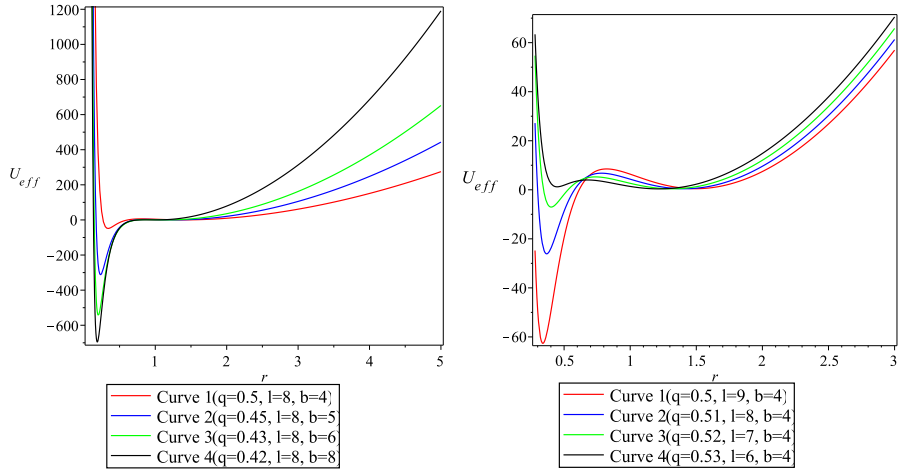


Figure 2: The plot of effective potential (U_{eff}) versus r . In the left panel, we fix $l = 8$ and varying q and b , respectively. In the right panel, we fix $b = 4$ and varying q and l , respectively.

We have drawn the graphs of effective potential U_{eff} vs radius r for different values of q , l and b in Figures **1-2**. It is apparent that the effective potential has a minimum near $r = 0.2$ corresponding to a stable circular orbit. However other circular orbits are also possible when the potential has maxima.

Differentiating the energy of the particle with respect to ρ by taking $\theta = \frac{\pi}{2}$, we obtain

$$\begin{aligned} \frac{dU_{\text{eff}}}{d\rho} &= \frac{2e^{\frac{2\alpha^2}{\rho}}(b^2\rho^5 + 2\alpha^2b^2\rho^4 + (1 + 2lb)\rho^3 - (2\alpha^2 + 4lb\alpha^2)\rho^2 - 3l^2\rho)}{(e^{\frac{2\alpha^2}{\rho}} + 1)\rho^5} \\ &+ \frac{2(b^2\rho^5 + 2bl\rho^3 - 3l^2\rho)}{(e^{\frac{2\alpha^2}{\rho}} + 1)\rho^5} + \frac{2(b^2l^2 - l^2\rho)}{\rho^4}, \end{aligned} \quad (21)$$

$$\begin{aligned} \frac{d^2U_{\text{eff}}}{d\rho^2} &= -\frac{\left((b^2\rho^5 + 2\alpha^2b^2\rho^4 + (1 + 2lb)\rho^3 - (2\alpha^2 + 4lb\alpha^2)\rho^2 - 3l^2\rho)\right)}{\rho^5(e^{\frac{2\alpha^2}{\rho}} + 1)^2} \\ &\times \left(\frac{4\alpha^2e^{\frac{2\alpha^2}{\rho}}}{\rho^2}\right) + \left(e^{\frac{2\alpha^2}{\rho}}(5b^2\rho^4 + 8\alpha^2b^2\rho^3 + 3(1 + 2lb)\rho^2 - 2(2\alpha^2 + 4lb\alpha^2)\rho - 3l^2)\right)\left(\rho^5(e^{\frac{2\alpha^2}{\rho}} + 1)^2\right)^{-1} \\ &+ \left((e^{\frac{2\alpha^2}{\rho}} + 1)^2(2l^5b^2 - 4l^2\rho) - \frac{8\alpha^2e^{\frac{2\alpha^2}{\rho}}}{\rho^2}(e^{\frac{2\alpha^2}{\rho}} + 1)(l^2b^2\rho - l^2\rho^2)\right)\left(\rho^5(e^{\frac{2\alpha^2}{\rho}} + 1)^2\right) - ((5\rho^4(e^{\frac{2\alpha^2}{\rho}} + 1)^2 - 4\rho^3\alpha^2e^{\frac{2\alpha^2}{\rho}}(e^{\frac{2\alpha^2}{\rho}} + 1))2e^{\frac{2\alpha^2}{\rho}}(b^2\rho^5 + 2\alpha^2b^2\rho^4 + \rho^3(1 + 2lb) - (2\alpha^2 + 4lb\alpha^2)\rho^2 - 3l^2\rho))(\rho^{10}(e^{\frac{2\alpha^2}{\rho}} + 1)^4)^{-1} \\ &- \left((5\rho^4(e^{\frac{2\alpha^2}{\rho}} + 1)^2 - 4\rho^3\alpha^2e^{\frac{2\alpha^2}{\rho}}(e^{\frac{2\alpha^2}{\rho}} + 1))(2l^5b^2 - 2l^2\rho)\right)((e^{\frac{2\alpha^2}{\rho}} + 1)^2\rho^9)^{-1} \\ &- 2\left((5\rho^4(e^{\frac{2\alpha^2}{\rho}} + 1)^2 - 4\rho^3\alpha^2e^{\frac{2\alpha^2}{\rho}}(e^{\frac{2\alpha^2}{\rho}} + 1))(2l^5b^2\rho^2 + b^2\rho^4 - 3l^2)\right)(\rho^9(e^{\frac{2\alpha^2}{\rho}} + 1)^4)^{-1} + \frac{2(5b^2\rho^4 + 6bl\rho^2 - 3l^2)}{\rho^5(e^{\frac{2\alpha^2}{\rho}} + 1)^2}. \end{aligned} \quad (22)$$

Here, l and b have these two relations given in Eqs.(21) and (22). After

collision and for $\theta = \frac{\pi}{2}$ and $\dot{\rho} = 0$, the energy given in Eq.(20) takes the form

$$E^2 = f(\rho) \left(1 + \rho^2 \left(\frac{l + \rho v_{\perp}}{\rho^2} - b \right)^2 \right). \quad (23)$$

Solving equation (23) for v_{\perp} , we get the escape velocity for the charge particle as

$$v_{\perp} = \frac{\rho \sqrt{E^2 - f(\rho)} + b\rho^2 - l}{\rho}, \quad (24)$$

where $f(\rho) = \left(1 - \frac{1}{\rho} \left(\frac{2}{e^{\frac{2\alpha^2}{\rho}} + 1} \right) \right)$. We have drawn the graphs of escape velocity v_{\perp} vs radius r for different values of E , q , l and b in Figures **3-5**. In left panel of Figure **3**, we are comparing the escape velocity for different values of energy, magnetic field and angular momentum by fixing the electric charge. We can observe that the possibility of the particle to escape is higher for large values of energy, magnetic field and angular momentum. In right panel of Figure **4**, we are fixing the angular momentum and varying total energy, magnetic field and electric charge. We can see that the behavior of escape velocity in Figure **4** is somehow similar to Figure **3**. From Figure **4** and **5**, we are observing the behavior of the particle to escape from the vicinity of the black hole under the effect of magnetic field. We can see from the figures that greater the strength of magnetic field the possibility for a particle to escape is more. Hence, we can conclude that the key role in the transfer mechanism of energy is played by the magnetic field which is present in the accretion disc to the particle for escape from the vicinity of black hole. This is in agreement with the result of [54, 55]. However, it is important to mention here that the trajectories of escape velocity is entirely different from [41] due to presence of exponential term in the regular blackhole metric.

4 Center of mass energy of colliding particles

- **Without magnetic field:**

The center of mass energy for colliding particle is define as

$$E_c = \sqrt{2}m_0(1 - g_{\mu\nu}u^{\mu}u^{\nu})^{\frac{1}{2}}. \quad (25)$$

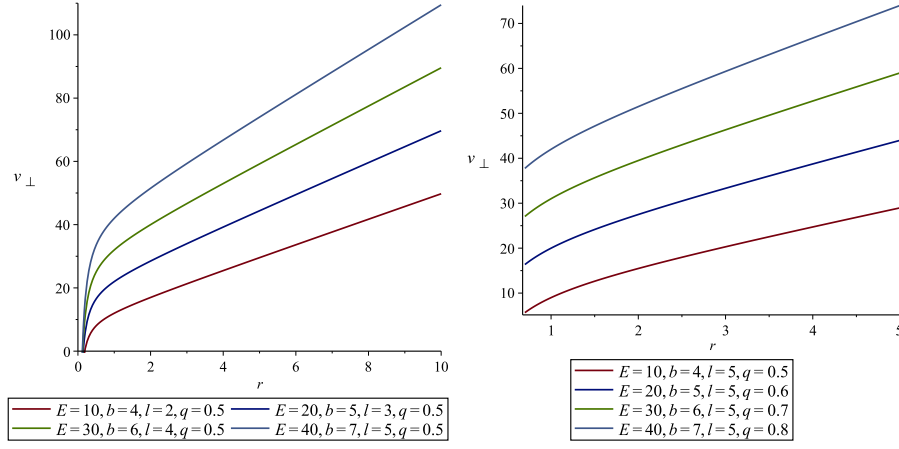


Figure 3: The plot of escape velocity (v_{\perp}) versus r . In the left panel, we fix $q = 0.5$ and varying l and b , respectively. In the right panel, we fix $l = 10$ and varying q and b , respectively.

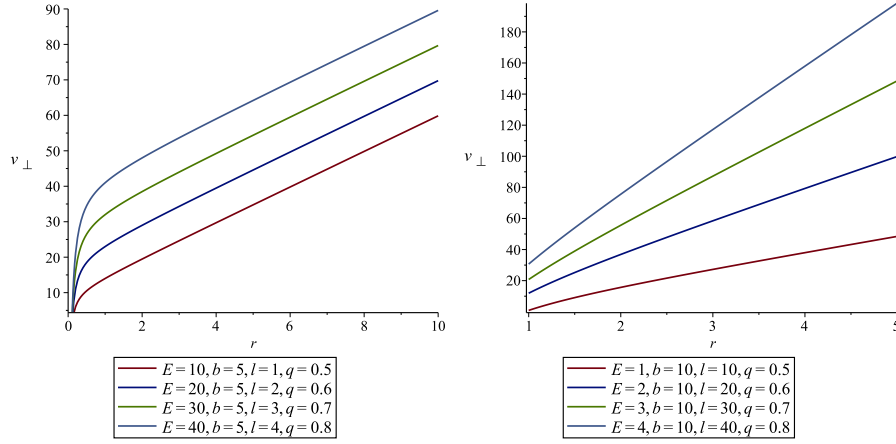


Figure 4: The plot of escape velocity (v_{\perp}) versus r . In the left panel, we fix $q = 0.5$ and varying l and b , respectively. In the right panel, we fix $b = 10$ and varying q and l , respectively.

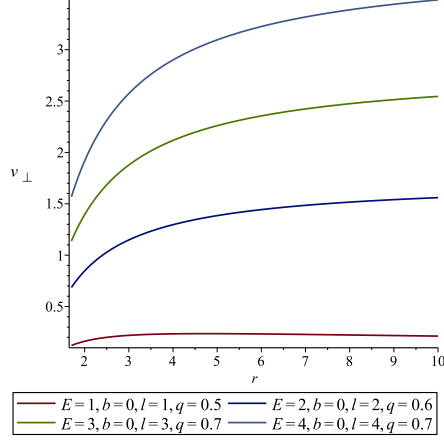


Figure 5: The plot of escape velocity (v_{\perp}) versus r by fixing $b = 10$ and varying q and l , respectively.

where u^{μ} is the 4-velocity of each particles. Using the values of Table 1 in Eq.(25), we have

$$E_c = \sqrt{2}m_0 \left(2 + \left(\frac{L_1^2 + L_2^2}{2r^2} \right) \left(\frac{E^2 + f(r)}{E^2} \right) + L_1 L_2 \left(\frac{f(r)L_1 L_2 - 2r^2 E^2}{2r^4 E^2} + \frac{f(r)}{2E^2} \right) \right)^{\frac{1}{2}}. \quad (26)$$

We want to obtain the CME near the horizon, where $f(r) = 0$), and we have

$$E_c = 2m_0 \left(2 + \frac{(L_1 - L_2)^2}{4r_h^2} \right)^{\frac{1}{2}}. \quad (27)$$

- **With magnetic field:**

By the normalization condition

$$\dot{r}^2 = f(r) \left(1 + r^2 + \left(\frac{L_z}{r^2} - B \right)^2 \right), \quad (28)$$

the center of mass energy takes the form

$$E_c = \sqrt{2}m_0 \left(2 + (L_1^2 + L_2^2) \left(\frac{f(r) + E^2}{2E^2 r^2} + \frac{B^2 f(r)}{2E^2} \right) - (L_1^2 + L_2^2) \right)$$

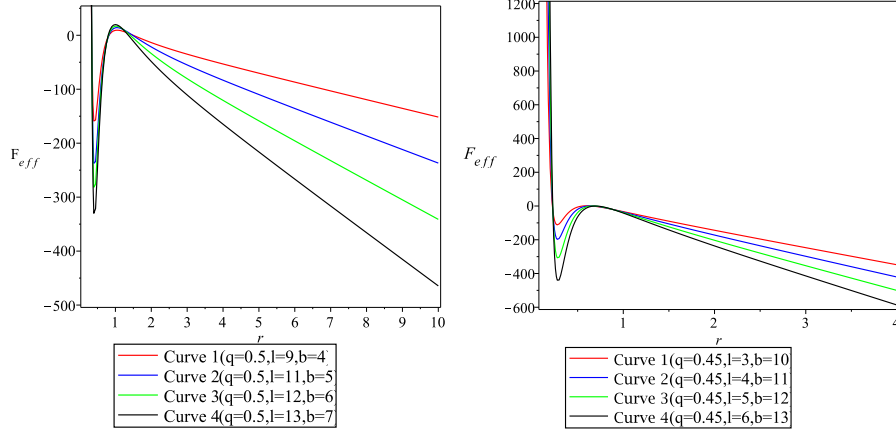


Figure 6: The plot of effective force (F_{eff}) versus r by fixing $q = 0.5$ (in the left panel) and $q = 0.45$ (in the right panel) while varying l and b .

$$\begin{aligned}
& \times \left(\frac{(f(r) + E^2)B^2}{E^2} + \frac{B^3 r^2 f(r)}{E^2} \right) + L_1 L_2 \left((L_1 L_2 + 2B r^2 (L_1 \right. \\
& + L_2) + 4B^2 r^4) \frac{f(r)}{2E^2 r^4} - \frac{1}{r^2} \Big) + \left(B^4 r^2 \left(\frac{E^2 + f(r)}{E^2} \right) + \frac{f(r)}{2E^2} \right. \\
& \left. \left. + \frac{f(r) r^4 B^4}{4E^2} \right) \right)^{\frac{1}{2}}. \tag{29}
\end{aligned}$$

The CME near horizon becomes

$$E_c = 2m_0 \left(1 + \frac{(L_1 - L_2)^2}{4r_h^2} - \left(L_1 + L_2 - B r_h^2 \right) \frac{B}{2} \right)^{\frac{1}{2}}. \tag{30}$$

5 Effective force

For a particle moving in the electromagnetic field in a flat background, the effective force is determined by the Lorentz force. In a curved background, the same force acting on the particle can be obtained by the derivative of effective potential as [43]

$$F_{\text{eff}} = -\frac{1}{2} \frac{dU_{\text{eff}}}{dr} = -e^{\frac{2\alpha^2}{r}} (b^2 r^5 + 2\alpha^2 b^2 r^4 + (1 + 2lb)r^3 - (2\alpha^2 + 4lb\alpha^2)r^2$$

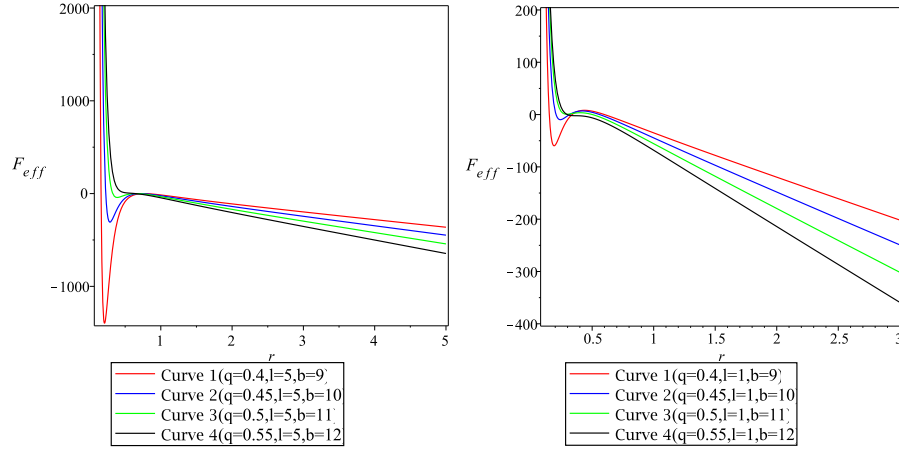


Figure 7: The plot of effective force (F_{eff}) versus r by fixing $l = 5$ (in the left panel) and $l = 1$ (in the right panel) while varying q and b .

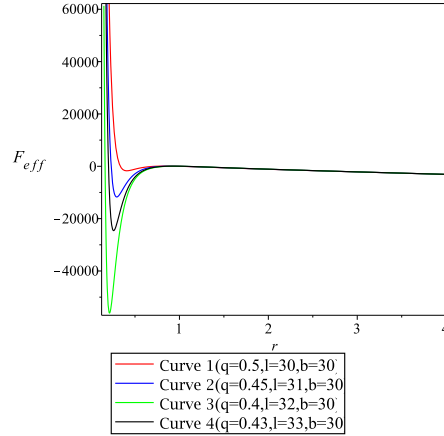


Figure 8: The plot of effective force (F_{eff}) versus r by fixing $b = 30$ and varying q and l .

$$\begin{aligned}
& - 3l^2r)((e^{\frac{2\alpha^2}{r}} + 1)r^5)^{-1} - \frac{(b^2r^5 + 2blr^3 - 3l^2r)}{(e^{\frac{2\alpha^2}{r}} + 1)r^5} - (b^2l^2 \\
& - l^2r)r^{-4}.
\end{aligned} \tag{31}$$

For rotational angular variable [43], we obtain

$$\frac{d\phi}{d\tau} = \frac{L_z}{r^2} - b \tag{32}$$

The Lorentz force on the particle is repulsive if $L_z > br^2$ and the Lorentz force is attractive if $L_z < br^2$. We have drawn the graphs of effective force F_{eff} vs radius r for different values of α , l and b in Figures 6-8. It is apparent from these figures that the effective force acting on the charged particles will be small as the distance from the black hole increases and vice versa if the distance shrinks. Thus at large distance, black hole's gravity and the electromagnetic field will have diminishing effects on the particles.

6 Concluding Remarks

In this work, first we have considered the regular black hole metric with non-linear electromagnetic source and studied the effective potential, escape velocity and energy of charged particle in presence of magnetic field. Next, we have investigated the center of mass energy of two colliding particles with and without magnetic field. Finally we found the effective force and analyzed the graphical representations of effective potential, escape velocity and effective force.

Although the black hole is regular (non-singular at all spacetime points), the geometry of spacetime outside the black hole's horizon should be similar to that of conventional black holes. Moreover the particles moving around such black holes should not feel the absence of the singularity. The present study also confirms the same features. Charged particle after collision can escape if the effective force acting on it is weaker. Hence, we can conclude that the key role in the transfer mechanism of energy is played by the magnetic field which is present in the accretion disc to the particle for escape from the vicinity of black hole. This is in agreement with the result of [54, 55]. However, it is important to mention here that the trajectories of escape velocity is entirely different from [41] due to presence of exponential term in the regular black hole metric.

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